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From restricted to full omniscience

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Abstract: **Some, notably Peter van Inwagen, in order to avoid problems with free will and omniscience, replace the condition that an omniscient being knows all true propositions with a version of the apparently weaker condition that an omniscient being knows all knowable true propositions. I shall show that the apparently weaker condition, when conjoined with uncontroversial claims and the logical closure of an omniscient being's knowledge, still yields the claim that an omniscient being knows all true propositions.**

Introduction

Peter van Inwagen has argued that God does not know what we shall freely do.[1] But van Inwagen does not embrace open-future views on which there is no fact of the matter about what we shall freely do. There is such a fact, but on van Inwagen's view, God does not know it. Nonetheless, van Inwagen holds that God is omniscient, because, roughly speaking, God knows every true proposition that *can* be known. I shall show, however, that the statement that God knows every knowable truth, as well as the precise statement of van Inwagen's 'tentative' account, have the consequence that God knows every true proposition, as long as we assume that God's knowledge is closed under conjunction and tautological consequence, together with some uncontroversial claims and, in the case of van Inwagen's account, the non-triviality claim that the account requires an omniscient being to now know some contingent truths.

'God knows all truths that it is possible to know'

Consider the 'restricted-omniscience' claim that God knows all truths (i.e. true propositions) that it is possible to know, and that there is a fact of the matter about what we will freely do. I will show that, when uncontroversial claims are conjoined, it follows that God knows what we shall freely do. I shall start with an informal argument showing that this in a special case.

Let p be the proposition that I will mow the lawn tomorrow, and suppose that this is true. Let q be the proposition that Barack Obama is now President of the United States. Let r be the proposition that I will mow the lawn tomorrow or Obama is not now President. Since p is true, r is true. Moreover, r is a proposition that can be known. For there is a world w^* where Obama is not now President and where God knows that Obama is not now President. Since God surely knows the tautological consequences of everything He knows, in w^* God knows that I will mow the lawn tomorrow or Obama is not now President, since He knows the second disjunct. Therefore, r is a true proposition that can be known. Hence, by the restricted-omniscience thesis that God knows all knowable truths, God knows r. But God surely knows that Obama is now President. Thus, God knows both (a) that I will mow or Obama is not President, and (b) that Obama is now President. Hence God also knows the tautological consequence of this, namely that I will mow. QED.

We can now formally state the three assumptions of this argument. Note that the first assumption in this statement is the restricted-omniscience claim, but slightly weakened to say not that God knows every true proposition that someone can know, but to say that God knows every true proposition that *God* can know – if the weaker claim, together with uncontroversial theses, entails full omniscience, obviously so does the stronger.

(1) $(p)[(p \mathbin{\&} \Diamond$ (God knows $p)) \supset$ God knows $p]$. [restricted omniscience]

(2) $\Box(p)(q)(r)[($God knows p & God knows q & r is a tautological consequence of $(p\&q)) \supset$ God knows $r]$. [closure]

(3) $(\exists p)[$God knows p & $\Diamond$ (God knows $\sim p)]$. [knowledge of one contingency]

Restricted omniscience is the hypothesis we are examining, hence there is no controversy. And claim (3) is obviously true when we let p be the proposition that Obama is President.

The only claim that might be questioned is closure. After all, closure is false if God is replaced with a finite knower, because probabilities can dwindle, so that p and q could each individually have epistemic probabilities that are above the threshold needed for knowledge, while their conjunction could fall below the threshold.

However, closure is not, in fact, controversial in the case of God. God's knowledge is *certain*. God cannot have false beliefs. This fact is central to the arguments of Pike[2] and van Inwagen for the alleged conflict between free will and God's knowing what we will freely do, and hence will be accepted by those who are driven by such arguments to restricted omniscience.

Besides, if we admitted that God's beliefs do not need to be certain, we would have a different route to the conclusion that God knows whether I will freely mow the lawn. If God can have false beliefs, God can have probabilistic knowledge.

Now, there is a possible world where it is possible to have probabilistic knowledge that I will freely mow the lawn. For instance, in a world where the laws of nature ensure that I will tomorrow face a choice between mowing the lawn or dying a horrible death at the jaws of an enraged bear, the probability of my mowing the lawn is so high that a God who can have probabilistic knowledge would know that I will mow the lawn. But then the claim that I will mow the lawn will be a claim that is both true and knowable by God, and hence by restricted omniscience, God would *actually* know the claim, even though in the actual world the claim is not overwhelmingly probable.

Now I will show that it follows from (1)–(3) that God knows all true propositions.

(4) Suppose p is true. [assumption for general conditional proof]
(5) Let q be a proposition such that God knows q and such that $\Diamond$ (God knows $\sim q$). [From (3) by existential instantiation]
(6) $\Box$(God knows $\sim q \supset$God knows $p \vee \sim q$). [special case of closure]
(7) $\Diamond$(God knows $\sim q$). [from (5)]
(8) $\Diamond$ (God knows $p \vee \sim q$). [from (6) and (7)]
(9) $p \vee \sim q$ is true. [from (4)]
(10) God knows $p \vee \sim q$. [from (1), (8), and (9)]
(11) God knows q. [from (5)]
(12) God knows p. [from (10), (11), and closure]
(13) Therefore, $(p)(p$ is true$\supset$God knows $p)$. [from (12) by general conditional proof]

QED.

van Inwagen's formulation

According to van Inwagen's 'tentative' account, x is (restrictedly) omniscient if and only if the following three conditions hold for every time t:

(14) x is able at t to consider or hold before its mind 'simultaneously' and in complete detail every possible world.
(15) For every set of possible worlds that contains the actual world and is such that it is possible (for any being) to know at t of that set that it contains the actual world, x knows at t of that set that it contains the actual world. (Here 'the actual world' is a definite description, a non-rigid designator of the world that happens to be actual. …).
(16) x's knowledge is closed under entailment (that is, if x knows that p, and if the proposition that p entails the proposition that q, then x knows that q) and x believes only what x knows.[3]

Say that a being is 'weakly logically omniscient' if and only if its knowledge is closed under entailment in the above sense.

As an addition to van Inwagen's account, I will add one further assumption, namely that the knowledge of an omniscient being is closed under conjunction:

(17) If x knows p and x knows q, then x knows $p\&q$.

The justification for this assumption is the same as the justification of closure in the preceding section. Besides, unless one has worries about dwindling probabilities, the intuitions supporting (16) also support (17). At the end of this section, though, I will consider what the argument would establish if (17) were denied.

Consider now the following technical assumption about a time t:

(18) There are sets S and T of possible worlds such that (a) S and T have no members in common, (b) the actual world is a member of S, (c) it is possible for an weakly logically omniscient being to know at t of S that it contains the actual world, and (d) it is possible for a weakly logically omniscient being to know at t of T that it contains the actual world.

If there is a set of all worlds, then (18) is really easy to show in the case where t is the present: Just let S be the set of all worlds where Obama is President and let T be the set of all worlds where Obama is not President. Then, (a)–(d) are clearly satisfied if God exists (I assume that God is at least restrictedly omniscient): for God knows of S that it contains the actual world, since the proposition that S contains the actual world is entailed by Obama's being President which God presumably knows, and in worlds in which God exists but Obama is not President, God knows that T contains the actual world for the same reason. (It seems quite fair to assume that God exists in this argument, both because van Inwagen accepts theism, and because the main interest in analysis of omniscience is the case of God.)

More generally, if there is a set of all worlds, then (18) will be satisfied at any time t at which there is a true proposition p such that a weakly logically omniscient being could know p at t and such that a weakly logically omniscient being could know $\sim p$ at t. If God exists, then every time since the creation of the very first contingent being B is like this, since at every such time, God knows that B was created, and yet in a world where God exists but did not create B, God knows that B was not created.

If there is no set of all worlds, then arguing for (18) will be technically harder, but we can do it as follows. I shall only run the argument in the case where t is the present time, to avoid complications. Start by observing that there either is or is not a set U such that the actual world is a member of U and

(19) It is possible for some being to know at t of U that U contains the actual world.

If there is no such set of worlds, then van Inwagen's definition of omniscience is compatible with the absurd claim that x could be omniscient, and yet right now know only necessary truths. For if there is no set U containing the actual world and satisfying (19), then (15) holds trivially even if x has no knowledge of contingent claims, and (14) and (16) do not require x to know any contingent claims.

Suppose now that God exists and there is a set U that satisfies (19) in the case where t is the present. Obviously, God knows that Obama is President, and by (15) and as God is (at least restrictedly) omniscient, it follows that God also knows that U contains the actual world. Assuming God's knowledge is closed under conjunction, God knows that Obama is President and U contains the actual world. Let S be the subset of U consisting of all the worlds in U at which Obama is President. Then conditions (b) and (c) in (18) are satisfied.

Now, there is surely a world w^* very much like ours, but where God knows that Obama is not President. It is then extremely plausible that there is also a set U* such that (19) holds at w^* with U* in the place of U, where t is the present (why should w differ from w^* in this respect?) Then, we can let T be the subset of U consisting of all the worlds in U* at which Obama is not President. By the same argument as above, it is true at w^* that God knows that T contains the actual world (remember that 'the actual world' is a definite description for van Inwagen, so that at w^* it is true that the actual world is w^*). Hence condition (d) in (18) is satisfied. Moreover, condition (a) is obvious since there is no world at which Obama is and is not President, and so (18) now holds.

I can now show that if x satisfies van Inwagen's definition of omniscience, and if the conjunctive closure condition (17) holds, then at any time t at which (18) holds, x knows all true propositions. I have also argued that if van Inwagen's definition is non-trivial in the sense that it requires God to know now some contingent truth, then (18) holds at the actual world right now. Hence, if van Inwagen's definition is satisfactory and restrictedly omniscient being now exists, that being now knows all true propositions.

The argument is as follows. Let x be restrictedly omniscient, and hence satisfy (14)–(16) (actually, we aren't going to need (14) at all), and assume the conjunctive closure condition (17). Let t be such that (18) holds at t. Let p be true at t. Let S and T be as in (18). From now on, for the sake of brevity, I shall be omitting 'at t' in various places in the argument by supposing that it is now t. Let $S_1 = T \cup \{w \mid w \in S$ & p is true at $w\}$. By choice of S and T, it is possible for someone who is weakly logically omniscient to know that the actual world is a member of T. Anyone weakly logically omniscient who knows that also knows that the actual world is a member of S_1 since T is a subset of S_1. Thus, it is possible for someone weakly logically omniscient to know that the actual world is in S_1. Moreover, the actual world is a member of S and p holds at the actual world, and so the actual world is, in fact, a member of S_1. Thus, by (15), x actually knows that the actual world is a member of S_1. But since S and T satisfy (18), it is possible for someone weakly logically omniscient to know that the actual world is a member of S. Since the actual world is a member of S, it follows by (15) that x knows that the actual world is a member of S. But that the actual world is a member of S entails that the actual world is not a member of T, since S and T have no members in common.

Therefore, x knows that the actual world is not a member of T, and x knows that the actual world is a member of S_1. Therefore, by (17), x knows that both the actual world is not a member of T and the actual world is a member of S_1. But since $S_1 = T \cup \{ w \mid w \in S \;\&\; p \text{ is true at } w \}$, the fact that the actual world is not a member of T and is a member of S_1 entails that the actual world is a member of $\{w \mid w \in S \;\&\; p \text{ is true at } w\}$, which in turn entails that p holds at the actual world, and hence also that p holds. Therefore, by (16), x knows p.

Hence, assuming that a restrictedly omniscient being satisfies the conjunctive closure condition (17), then at any time at which (18) holds, a restrictedly omniscient being knows every true proposition. Since I've argued that if van Inwagen's definition is to be plausible, we should assume that (18) holds, it follows that God now knows every true proposition, assuming God exists.

Suppose the conjunctive closure condition (17) is rejected. Assuming that we are talking of a time at which (18) holds (note though that one of the argumentative routes to (18) used (17)), we get stuck in the argument at the point where x knows that the actual world is not a member of T and x knows that the actual world is a member of $T \cup \{w \mid w \in S \;\&\; p \text{ is true at } w\}$. By van Inwagen's closure-under-entailment condition (16), x will know that p is true or the actual world is a member of T, since that is entailed by the claim that the actual world is a member of $T \cup \{ w \mid w \in S \;\&\; p \text{ is true at } w \}$. Let q be the proposition that the actual world is a member of T. Then, x knows $p \vee q$ and x knows $\sim q$. Let r be $\sim q$. Then, by (16) our restrictedly omniscient x knows $r \supset p$ and knows r, although, if p is a future contingent and x does not know future contingents, then x does not know p.

In other words, if the conjunctive closure condition (17) is rejected, what my argument establishes is this claim: There is a single proposition r (independent of p) such that (a) x knows r and (b) for every true proposition p it is the case that x knows $r \supset p$. This means that a restrictedly omniscient being *almost* knows every truth – for every truth the being knows a material conditional with that truth as its consequence and with a known truth as antecedent. It would be a very strange view of God that God did not, say, know that I will mow the lawn, but God knew r and knew that if (materially) r, then I will mow the lawn.

Furthermore, I suspect that such a view would not gain one anything over the classical view of unrestricted omniscience. The basic worry about the classical view expressed by Pike and van Inwagen is basically that if the state of the world prior to my choosing to mow the lawn entails that I will mow the lawn, then I cannot not mow the lawn. Suppose that prior to my choice whether to mow, a being whose beliefs cannot be false believes r and believes that if r, then I will mow the lawn. Then the state of the world prior to my choosing to mow the lawn does entail that I will mow the lawn, just as much as it would if we had a being whose beliefs could not be false that simply believed that I will mow the lawn.

Some final remarks

The basic problem with saying that a restrictedly omniscient being knows all knowable true propositions is that there are contingently true propositions q such that it is sometimes easy to know q when q is true and sometimes easy to know $\sim q$ when q is false, and then we can apply the restricted omniscience thesis to the disjunction of an arbitrary true proposition with $\sim q$.

Could a restricted omniscience theorist, perhaps, just rule disjunctions out of court? Maybe she could just say that God knows all knowable true non-disjunctive propositions, as well as all entailments of these.

However, there is a problem with that suggestion. The first is that disjunctive claims can be dressed up in all sorts of non-disjunctive ways. For instance, consider the following propositions:

(20) It is not the case that both: I will not mow the lawn tomorrow and Obama is President.

(21) If (in the material conditional sense) Obama is President, then I will mow the lawn tomorrow.

(22) The set of true propositions that are also members of { that I will mow the lawn tomorrow, that Obama is not President } is at least as large in cardinality as the set {Plantinga}.

(23) Were I to utter the sentence 'I will not mow the lawn tomorrow and Obama is President', I would be uttering a sentence that expresses a false proposition.

(24) The set $\{e \mid e$ is an event and (e is a mowing by me of the lawn tomorrow or e is Obama's being President today)$\}$ is non-empty.

Assuming I will mow the lawn tomorrow, each of these propositions is true and possibly known, and each of them is such that if God (or any sufficiently competent epistemic agent who knows that Obama is President) knew it, then God would know that I will mow the lawn tomorrow. Yet none of these is strictly speaking disjunctive in form.

We could try to say that (20) and (21) are disjunctive in form because they are tautologically equivalent to disjunctions. But of course *every* proposition is tautologically equivalent to a disjunction (e.g., p is always tautologically equivalent to $(p\&q) \vee (p\&\sim q)$). Or we might say that just as we should rule out disjunctions, so too we should rule out negations of conjunctions and material conditions. But that would still leave (22)–(24).

Suppose, though, that in some clever way we have excluded all propositions that are somehow disjunctive[4] from the scope of the claim that an omniscient being knows all true knowable propositions. But then we have excluded too much. For suppose that the laws of nature determine it to be the case that tomorrow I will freely choose whether to mow the lawn or bake a cake (or that an electron will go up or go down in the magnetic field). Then an omniscient being had better know that tomorrow I will choose to mow the lawn or tomorrow I will choose to bake a cake.

But this is a disjunction, and the definition of restricted omniscience constrained to exclude disjunctions does not appear sufficient to guarantee that an omniscient being will know this. One might worry that here we have an *exclusive* or – I won't both choose to mow the lawn *and* bake a cake, while we only excluded *inclusive* disjunctives from the scope of the claim that an omniscient being knows all true knowable propositions. But we had better exclude exclusive disjunctions just as much, as can be seen from consideration of the exclusively disjunctive proposition that Obama is not President today or (I will not mow the lawn tomorrow with Obama being President today), but not both.

Recent work on the Principle of Sufficient Reason[5] (PSR) has tended to show that apparent weakenings of the PSR end up entailing the full PSR, or at least a PSR strong enough to generate cosmological arguments. It appears that a similar claim holds in the case of certain weakenings of definitions of omniscience. If so, this fact provides argumentative support for the simpler and more traditional account of (propositional) omniscience according to which God knows all true propositions and cannot believe any that are not true.[6]

Notes

1. Peter van Inwagen 'What does an omniscient being know about the future?', in J. Kvanvig (ed.) *Oxford Studies in Philosophy of Religion*, I (Oxford: Oxford University Press, 2008), 216–230.
2. Nelson Pike 'Divine omniscience and voluntary action', *Philosophical Review*, **74** (1965), 27–46.
3. van Inwagen 'What does an omniscient being know', 224 (my numbering of conditions).
4. If one believes in truth-makers, one might try the following trick. Say that a proposition p is 'shifty' provided that there are two worlds w_1 and w_2 at each of which p is true, but p has a truth-maker at w_1 which does not exist in w_2. For instance, the proposition that I will mow the lawn tomorrow or Obama is President is shifty, since in some worlds it has the event of my mowing the lawn tomorrow as a truth-maker, while in other worlds it is still true, even though that event does not exist. Then one simply excludes all shifty propositions. I do worry, though, that excluding all shifty propositions will exclude just about every interesting proposition. For instance, take the proposition that George greeted Sarah. In one world this is made true by George's saying 'Hello' to Sarah while in another this is made true by his saying 'Hi' to her.
5. E.g. Alexander R. Pruss *The Principle of Sufficient Reason: A Reassessment* (New York: Cambridge University Press, 2006).
6. I am grateful to Jonathan Kvanvig and commenters on *prosblogion.ektopos.com* for discussions of these topics.